

Gaps on the flow of the simplified path-preference cellular automaton model

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The path-preference model is a cellular-automaton model to describe the dynamics of RNA polymerase II in transcription. The main difference from the standard traffic flow model is that it contains another preferential paths at some sites. In this paper, we propose an exact analysis for the simplest version of this model which consists on the minimal structure and removes stochastic factors. We find that the number of particles is dominant to the dynamics of this version and observed that there are not only expected phase shift but also several non-continuous gaps as the number of particles increases. By considering the behavior of limit cycles, we also determine the point where such gaps in the flow appear and the exact value of the flow.

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I. INTRODUCTION

Transcription is the creation process of messenger RNA from the DNA information and one of the fundamental cellular processes. In this process, RNA polymerase enzyme plays an important role [1, 2] but its dynamics is not yet fully understood. In the classical model, RNA polymerases move along DNA templates and synthesizing pre-mRNA by reading their information. This model is applicable only to the dynamics of prokaryotic cells because of its simplicity. That of eukaryotic cell is more complicated due to several factors. For example, DNA of prokaryotic cell consists not only exon but also intron [1] and the pre-mRNA synthesized in intron degrades when RNAPII reaches exon [3]. It is also observed that some sites of DNA can be modified by proteins and these modifications regulate transcriptional dynamics [3–6]. Furthermore, RNAPIIs in eukaryotic cell interact with each other cooperatively. Transcription factory [7, 8] is the model describing such behavior, where RNAPIIs cluster for efficiency of transcription [9–11].

The traffic flow models are the system consisting on self-driven particles which interact with each other by excluded volume effect and applied to analyze not only vehicular traffic but also several fields including life sciences. The dynamics of this model is expressed by differential equations or cellular automata [13–19] and the latter are preferred with regard to ease for computer simulations. The simplest cellular automaton model is also known as Wolfram’s elementary cellular automaton (ECA) rule 184 [20]. By virtue of the limiting procedure called “ultradiscretization” [21], this model is related to the Burgers equation, which is one of differential equation model.

Such discovery of transcription dynamics, development of traffic flow models and finer data of the dynamics of polymerase by virtue of the recent progress of the measurement technology yield several transcription models based on traffic flow models. Tripathi et al. suggested the model where RNAPII take several states and interact each other and discussed that their mean velocity and density [22, 23]. Ohta et al. suggested the traffic model of RNAPII with blockades which are regarded as the protein modification and discussed the response of RNAPII after stimulation and its time intervals [5]. A recent experiment indicates that there are several gaps in the distribution of the position of RNAPII [3], which cannot be explained by the existing traffic flow models. To understand such a phenomenon, Ohta et al. proposed the path-preference model [24], which accepts RNAPII near the DNA loop formation in the end of an exon to jump onto the top of the next exon. Such jumps are interpreted as three dimensional diffusion due to concentration of RNAPII in transcription.

It is known that the numerical simulation for this model in deterministic case shows several discontinuous gaps in the flow [25]. By analyzing orbits of the system in some special cases, we discover the reason why such gaps appear. In this paper, we discuss mathematical properties of the path-preference model including these gaps. In section 2, we briefly review the path-preference model. In section 3, we discuss the mathematical properties in the simplest case of

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FIG. 1: An example of states of the path-preference model. Boxes and balls express sites and particles, respectively, and the gray sites mean that they belong to the e -parts. Particles translate rightward in this picture.

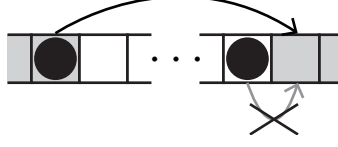


FIG. 2: The particle at the end of an e -part can jump to the the top of the next e -part stochastically if there are no particles. The particle at the end of an i -part cannot move to the top of the next e -part if a particle jumps there.

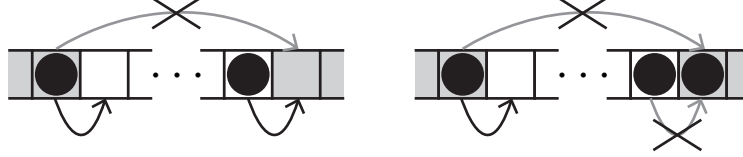


FIG. 3: If particles cannot jump for some reason (the destination is occupied or the jump is stochastically inhibited), the time evolutions of particles at these sites are dominated by the movement rule.

the path-preference model, which is corresponding to deterministic 1-exon and 1-intron one. Finally, we numerically discuss extensions of this model by introducing stochastic factor or considering more complex structure.

II. PATH-PREFERENCE CELLULAR AUTOMATON MODEL

A. Modeling

In this section, we briefly review the path-preference cellular automaton model. This system consists on a sequence of sites and several particles and each site can contain at most one particle. The sites are divided into two classes, e -part and i -part, and we assume that the first site belongs to e -part and the last one to i -part (See Fig. 1). The time evolution of the system is completed when all particles have moved by the following rules.

Except some special sites described later, a particle moves to the next site if there are no particles or stalls if there is. This rule is the same as the local update of the ECA rule 184 and means the excluded volume interaction of particles. We define that a particle which is on the end of the last i -part goes back the top of the first e -part, i.e., this system is periodic.

The difference from the standard traffic flow cellular automaton model is that the particle on the end of an e -part stochastically jumps to the top of the next e -part. The destination of the jump on the last e -part is the top of the first one due to the periodicity. In traffic flow models, this jump means the shortcut between these sites.

In the paper [24], *sites*, *sequences of sites*, e -, i -part and *particles* are expressed as bases, DNA, exon, intron and RNAPII, respectively, the orientation of the transition is that of transcription and the periodical boundary condition means that RNAPII which has finished its transcription on a DNA immediately starts again that on the next DNA. However, this model can be also applied to the dynamics of RNAPII which travels between gene area and non-gene area in a single exon.

In this model, the particle on the end of an e -part can jump to the top of the next one and that on the end of the previous i -part also can move there. Then, we should determine priority for these two actions to avoid collision. In this paper, we give the priority to jumping particles because of a mathematical reason described in later, i.e., if there is a particle jumping to an e -part, no particles can move there from the previous i -part (See Fig. 2 and 3).

B. mathematical representation

Based on the modeling above, we now write down the mathematical formulation of this model. We denotes N as the number of sites and each site is labeled as $1, \dots, N$ in series. Let $u_j^t \in \{0, 1\}$ ($j = 1, 2, \dots, N$, $t \in \mathbf{Z}_{\geq 0}$) be the number

of particles at the site j and time t (and set N sufficiently large for consistency of the definition below). The system contains $K(\geq 1)$ e - and i -parts and the number ϵ_k and ι_k ($k = 1, \dots, K$) satisfying $1 < \epsilon_1 < \epsilon_2 < \dots < \epsilon_K < N$ and $1 < \iota_1 < \iota_2 < \dots < \iota_K = N$ are the last site of k -th e - and i -part, respectively. We denote m_k^t ($k = 1, 2, \dots, K$) as the stochastic variables which takes 1 for probability p and 0 for probability $1 - p$, where p ($0 \leq p \leq 1$) is a constant.

For the simple expression of the dynamics, we employ the local flow $f(k, l)$, which is the number of transiting particles from site k to site l . Due to the time evolution rule, the movement of a particle to the next site is expressed as

$$f(j, j+1) = \min(u_j^t, 1 - u_{j+1}^t) \quad (1)$$

for $j \neq \epsilon_k, \iota_k$ ($k = 1, 2, \dots, K$).

Since we introduce the jump priority rule, the number of jumping particles from the end of k -th e -part to the top of $k+1$ -th one is

$$f(\epsilon_k, \iota_k + 1) = \min(u_{\epsilon_k}^t, 1 - u_{\iota_k+1}^t, m_k^t). \quad (2)$$

In the case $f(\epsilon_k, \iota_k + 1) = 1$, one has

$$f(\epsilon_k, \epsilon_k + 1) = 0 \quad (3)$$

$$f(\iota_k, \iota_k + 1) = 0 \quad (4)$$

and in the case $f(\epsilon_k, \iota_k + 1) = 0$,

$$f(\epsilon_k, \epsilon_k + 1) = \min(u_{\epsilon_k}^t, 1 - u_{\epsilon_k+1}^t) \quad (5)$$

$$f(\iota_k, \iota_k + 1) = \min(u_{\iota_k}^t, 1 - u_{\iota_k+1}^t). \quad (6)$$

Here, we consider the modulo of N for the index of the site number.

The time evolution of u_j^t is written in the form:

$$u_j^{t+1} = u_j^t + \Delta u_j^t, \quad (7)$$

where Δu_j^t is the subtraction of the number of outgoing particles at site j from that of incoming ones, which is expressed as a combination of some $f(k, l)$. Then, we obtain the time evolution rule:

$$u_{\epsilon_k}^{t+1} = u_{\epsilon_k}^t + f(\epsilon_k - 1, \epsilon_k) - f(\epsilon_k, \iota_k + 1) - f(\epsilon_k, \epsilon_k + 1) \quad (8)$$

$$u_{\iota_k+1}^{t+1} = u_{\iota_k+1}^t + f(\iota_k, \iota_k + 1) + f(\epsilon_k, \iota_k + 1) - f(\iota_k + 1, \iota_k + 2) \quad (9)$$

and

$$u_j^{t+1} = u_j^t + f(j-1, j) - f(j, j+1). \quad (10)$$

for other sites.

Obviously, by adding both hand sides for $j = 1, \dots, N$, one has $\sum_{j=1}^N u_j^{t+1} = \sum_{j=1}^N u_j^t$, which means the conservation law of the total number of particles. Furthermore, by introducing a new dependent variable $v_j^t := 1 - u_j^t$, the time evolution rule of v_j^t is the same as that of u_j^t under the replacement $j \rightarrow -j$, which means that the dynamics is the same as that of particle under the inverted direction when observing the states for each time in interchanging empty sites and occupied ones. Such a good symmetry does not hold if one gives priority to the movement.

We note that jump probability p is not constant but monotonously increasing as time passes in the original model proposed by Ohta et al. because RNAPIIs are more concerned as transcription started. We also note that the system is the same as the ECA rule 184 if jump probability p is equal to 0.

Fig. 4 shows an example of the time-space pattern of this system. By increasing the number of particles, traffic jams happens at the end of i -parts. To understand such a phenomenon, we consider the dynamics of the simplest case in the next section.

III. ANALYSIS OF THE SIMPLIFIED PATH-PREFERENCE MODEL

To construct the fundamentals of the mathematical analysis for the path-preference model, we consider the simplest case, where the system consists only on one e -part and one i -part and the jump probability $p = 1$. Now, we define N_e

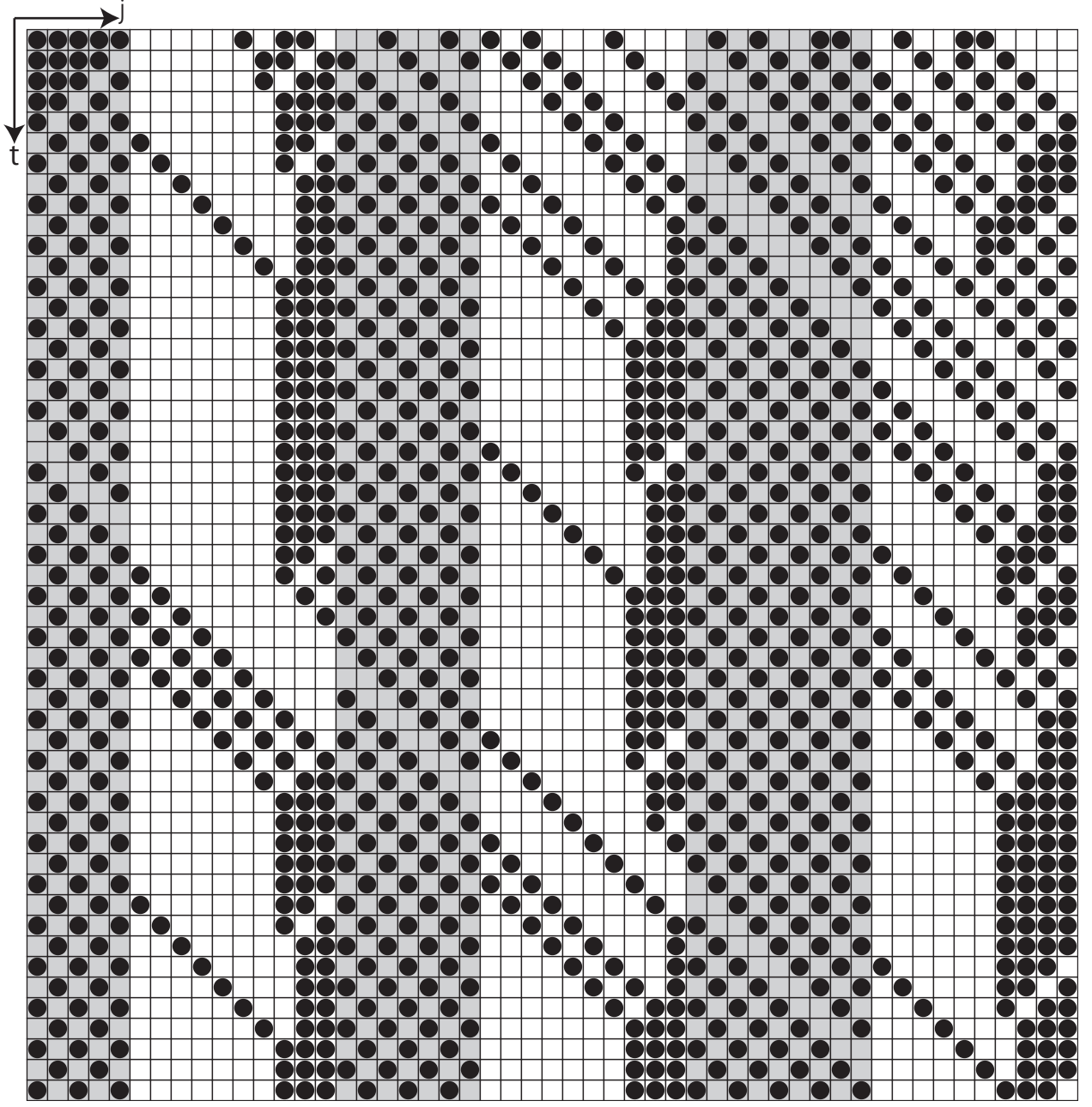


FIG. 4: An example of time evolution for the original path-preference model by setting $\epsilon_1 = 5$, $\epsilon_2 = 7$, $\epsilon_3 = 9$, $\iota_k = 10$ ($k = 1, 2, 3$) and $p = 0.8$.

and N_i as the number of sites in the e -part and the i -part (then, $N = N_e + N_i$), i.e., the sites from $j = 1$ to $j = N_e$ belong to the e -part and $j = N_e + 1$ to $j = N$ to the i -part. By this simplification, the particle on the end of the e -part must go back to its top if there are no particles. The mathematical representation of this simplest version is shown in Appendix A. We note that the orbit which started from an arbitrary initial value is finally attracted by a limit cycle because the time evolution rule is deterministic and the number of states are finite. Then, the behavior of the system is finally characterized by limit cycles.

A. General aspect

We first consider the behavior of this system by focusing on the number of particles M . In the case where M are sufficiently small, all particles are finally trapped by the e -part once they arrived there for most of initial values due to the time evolution rule and such behavior can be kept for $M \leq N_e/2$. If $M > N_e/2$, the e -part cannot hold all particles and overflowed ones are emitted to the i -part. In other words, the traffic on the e -part is always optimized for maximizing efficiency of the traffics. In particular, if N_e is even, particles in the e -part generate a closed flow which prohibits other particles to join and those in the i -part never come back to the e -part and finally cause the unsolvable traffic jam at the end of the i -part. If N_e is odd, there is at least one area which satisfies $u_j^t = 0$ and $u_{j+1}^t = 0$ in the e -part for any limit cycles. At space-time pattern, this area looks like a notch, which permits the particles at the i -part to move back to the e -part and helps the dynamics be richer. Hereafter, we mainly consider the case where N_e is odd. We stress that limit cycles are not uniquely determined only by given M but depend on the configuration of particles at the initial state. Most of initial states run the orbit which belong to a general class discussed here but some of them to other more complicated classes discussed later.

B. Analysis focusing on flow

The value “flow”, which is defined as the number of moving particles per a site, is a powerful tool for analyzing the behavior of the general traffic models. However, it is not naively applicable for this model because there is an ambiguity of the distance for the jump. To match a model with experiments, we should take this distance properly. For example, it should be 1 in the figure of 8-shape traffic model and it is taken as the length of the hurdled intron in the original path-preference model. In this paper, we consider the flow of each part instead of whole flow to avoid such not so essential discussion and focus on mathematical one. We also ignore the movement between two parts. By denoting J_e and J_i as the flow of the e -part and the i -part respectively, they are written in

$$J_e = \frac{1}{N_e - 1} \sum_{j=1}^{N_e-1} f(j, j+1) \quad (11)$$

$$J_i = \frac{1}{N_i - 1} \sum_{j=N_e+1}^{N-1} f(j, j+1). \quad (12)$$

We define the time average of value $X(= J_e, J_i)$ by

$$\langle X \rangle = \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} X. \quad (13)$$

Now, remember that arbitrary orbits are attracted by limit cycles. By introducing the period of a limit cycle T_0 , one can rewrite

$$\langle X \rangle = \frac{1}{T_0} \sum_{t=t_0}^{t_0+T_0-1} X \quad (14)$$

for sufficiently large t_0 . We note that the period depends of the initial states, i.e. $T_0 = T_0(\{u_j^0\}_{j=1}^N)$

FIG. 5 and 6 are graphs relating between the flow $\langle J_e \rangle$, $\langle J_i \rangle$ and the number of particle M for fixed N_e and N_i . One can find that not only phase shift around $M = N_e/2$ discussed above but also several non-continuous gaps appear in the flow in the i -part. One can also observe that the flow in the i -part can take several value for some M . We note that the flow is symmetric with $M = N_e/2$ because the dynamics of $v_j^t = 1 - u_j^t$ is the same as that of u_j^t . To discuss the reason of such phenomena including the exact point of gaps and flow of each part, we focus on several representative limit cycles.

C. Limit cycle: class 1-0

We first consider the case where N is even and $M = N/2$. In this case, arbitrary orbits are finally attracted by the limit cycle $u_j^t = t + j \pmod{2}$ with period 2, which is the same as that of the ECA rule 184. Therefore, the exact

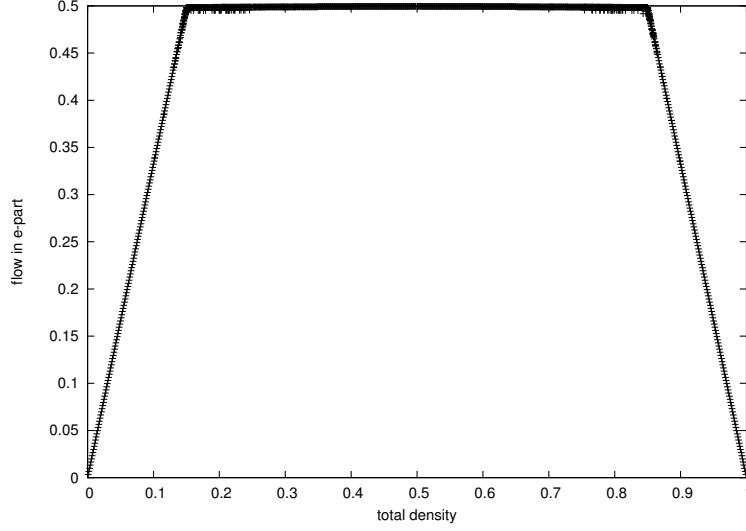


FIG. 5: The graph of the flow in the e -part $\langle J_e \rangle$ for several initial state for each M where $N_e = 301$ and $N_i = 700$. The horizontal axis is the density of the particles M/N (the number of particles normalized by the system length). The vertical axis is $\langle J_e \rangle$.

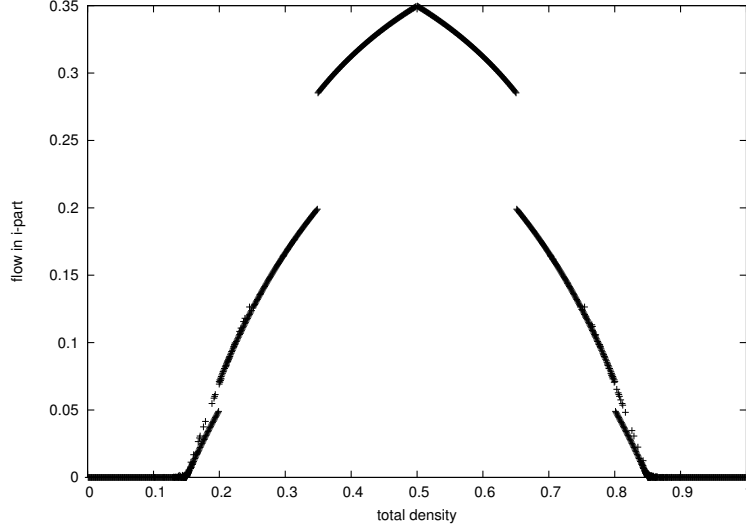


FIG. 6: The graph of the flow in the i -part $\langle J_i \rangle$ for several initial state for each M where $N_e = 301$ and $N_i = 700$. The horizontal axis is the density of the particles M/N (the number of particles normalized by the system length). The vertical axis is $\langle J_i \rangle$.

flow is

$$\langle J_e \rangle = \langle J_i \rangle = \frac{1}{2}. \quad (15)$$

D. Limit cycle: class 1-1

We next consider the case $M < N/2$ and the initial state $u_j^t = 1$ ($3 \leq j \leq 2M - 1$, j is odd) and $u_j^t = 0$ for other sites. The notch at the e -part is first at the top, moves to the end as time evolves and finally arrives at there in $t_0 = N_e - 1$. The head of particles at the i -part arrives the end in $t_1 = N - 2M - 1$. If $t_0 \geq t_1$, the particles at the i -part form a jam until $t = t_0 + 1$ because a particle occupies the site $j = 0$ or that at $j = N_e$ jumps there. Then, the head particle can move to the top of the e -part at $t = t_0 + 2$. At the next time, a particle is at $j = N_e$ again and $j = 0$ is occupied by the particle which moved from $j = N$. Then, the particle at $j = N_e$ moves to $j = N_e + 1$ and

both $j = 0$ and $j = N_e$ become unoccupied. At the next time ($t = t_0 + 4$), the particle at $j = N$ moves to $j = 0$ and the particle at $j = N_e - 1$ moves to $j = N_e$. Repeating this $M - (N_e - 1)/2$ times, all particles forming the jam at the end of the i -part come back to the e -part and net $M - (N_e - 1)/2$ particles at the e -part are emitted to the i -part and particles occupy at $j = 3, 5, \dots, N_e, N_e + 2, \dots, 2M - 1$ at $t = 2M + 1$, which is the same as the initial state. Therefore, this orbit forms a limit cycle whose period is $T = 2M + 1$. Fig. 7 shows an example of such limit cycles.

Now, the time average of the flow at the i -part is expressed as the number of particles emitted to the i -part per a period and it is the same as that in the i -part which are at the i -part at $t = 0$. Then, the flow at the i -part is expressed as

$$\langle J_i \rangle = \frac{2M - N_e + 1}{2(2M + 1)} \quad (16)$$

and the similar discussion yields the exact value of the flow at the e -part by

$$\langle J_e \rangle = \frac{2M}{2(2M + 1)}. \quad (17)$$

The condition $t_0 \geq t_1$ is written in

$$\frac{N_i}{2} \geq M, \quad (18)$$

which is the necessary condition where orbits can be attracted this limit cycle.

E. Limit cycle: class 1-2

If $t_0 < t_1$, the head of particles at the i -part does not reach to the end at $t = t_0$. Then, the notch once goes back to $j = 0$ and moves to the end again. The next time when it arrives there is at $t'_0 = 2N_e + 1$. If $t'_0 \geq t_1$, particles which formed the jam at the end of the i -part move back to the e -part at $t = t'_0 + 2$ and by the same discussion as above, the period is $T_0 = 2M + N_e + 1$ and the flow of each part is written in

$$\langle J_i \rangle = \frac{2M - N_e + 1}{2(2M + N_e + 1)} \quad (19)$$

$$\langle J_e \rangle = \frac{2M + N_e - 1}{2(2M + N_e + 1)} \quad (20)$$

and the condition $t'_0 \geq t_1 > t_0$ is

$$\frac{N_i - N_e}{2} \leq M < \frac{N_i}{2}. \quad (21)$$

We next consider the initial state $u_j^t = 1$ ($3 \leq j \leq 2m_1 - 1$, $2m_1 + 2N_e + 1 \leq j \leq 2m_1 + 2m_2 + 2N_e + 1$, j is odd) and $u_j^t = 0$ for other sites, where m_1 and m_2 are positive number satisfying $m_1 + m_2 + (N_e - 1)/2 = M$. In this initial state, particles of the i -part are split in two blocks. The head particle at the i -part arrives at $j = N$ in $t_1 = N - 2m_1 - 2m_2 - 2N_e - 1$ and the notch arrives at $t_0 = N_e - 1$. If $t_1 < t_0$, by the same discussion as above, the head block of the i -part moves to the e -part and m_2 particles of the e -part are emitted to the i -part. Then after passing $t = 2m_2 + t_0 + 1$, the state is $u_j^t = 1$ ($3 \leq j \leq 2m_1 - 1$, $2m_1 + 2N_e + 1 \leq j \leq 2m_1 + 2m_2 + 2N_e + 1$, j is odd) and $u_j^t = 0$ for other sites, which is the same as the initial state by interchanging m_1 and m_2 . Then, by repeating this procedure again, we can show that the orbit is a limit cycle whose period is $T_0 = 2M + N_e + 1$ and flow is

$$\langle J_i \rangle = \frac{m_1 + m_2}{T_0} = \frac{2M - N_e + 1}{2(2M + N_e + 1)} \quad (22)$$

$$\langle J_e \rangle = \frac{2M + N_e - 1}{2(2M + N_e + 1)}, \quad (23)$$

which are the same as (19) and (20). Then, the first case is a special one of this obtained by setting $m_2 = 0$.

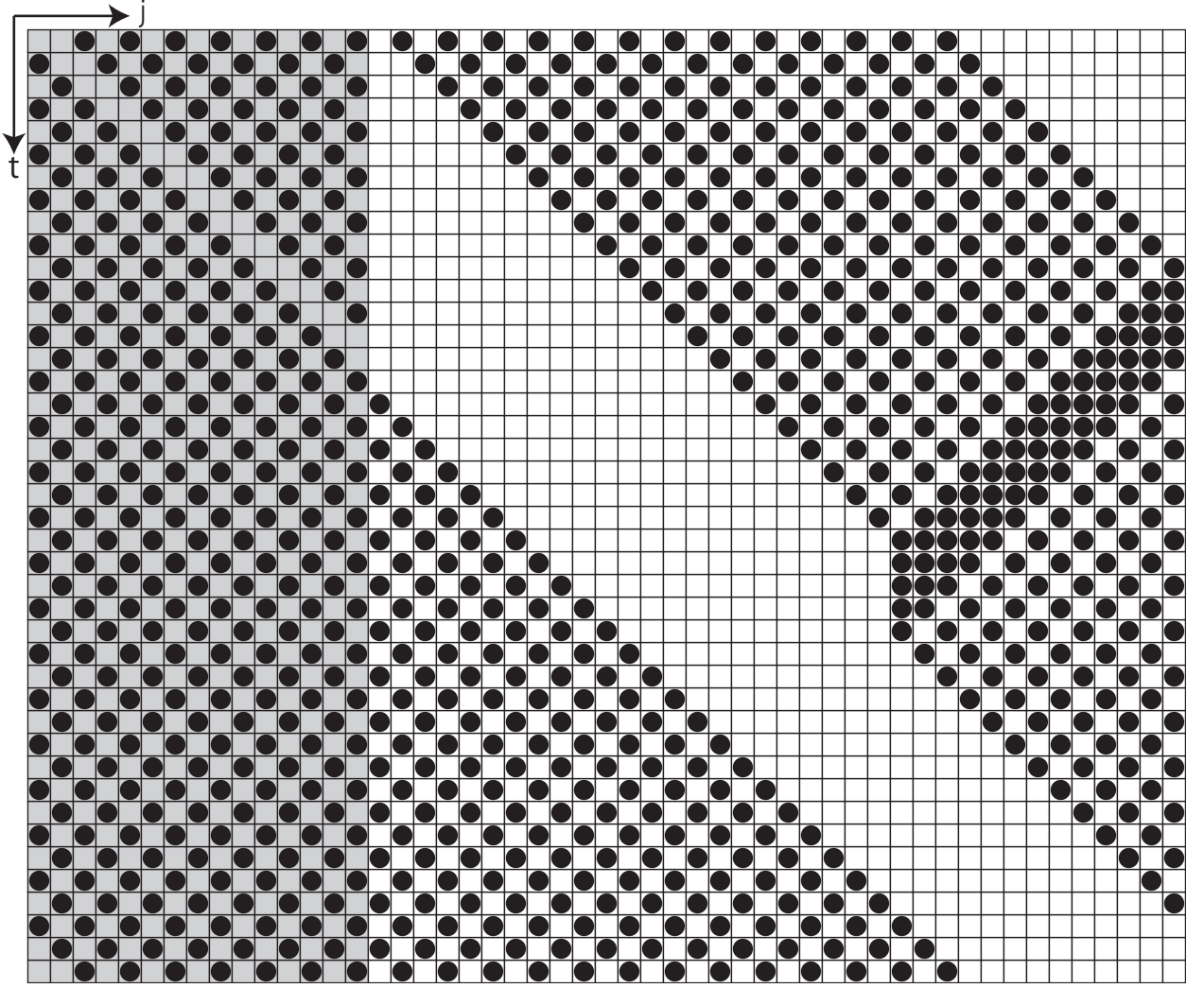


FIG. 7: An orbit of limit cycle class 1-1 for 1 period. The parameters are $N_e = 15$, $N_i = 36$, $M = 20$.

F. Limit cycle: class 1- γ

By generalizing this discussion, we can show that the initial state $u_j^t = 1$ ($3 \leq j \leq 2m_1 - 1$, $2m_1 + 2N_e + 1 \leq j \leq 2m_1 + 2m_2 + 2N_e + 1$, \dots , $2m_1 + \dots + 2m_\gamma + 2(\gamma - 1)N_e + 1 \leq j \leq 2m_1 + \dots + 2m_\gamma + 2(\gamma - 1)N_e + 1$ and j is odd) and $u_j^t = 0$ for other sites creates a limit cycle whose properties are

$$T_0 = 2M + (\gamma - 1)N_e + 1 \quad (24)$$

$$\langle J_i \rangle = \frac{2M - N_e + 1}{2(2M + (\gamma - 1)N_e + 1)} \quad (25)$$

$$\langle J_e \rangle = \frac{2M + (\gamma - 1)N_e - (\gamma - 1)}{2(2M + (\gamma - 1)N_e + 1)} \quad (26)$$

for $\gamma \geq 0$ and the necessary condition to attract this limit cycle is

$$\frac{N_i - (\gamma - 1)N_e}{2} \leq M < \frac{N_i - (\gamma - 2)N_e}{2} \quad (27)$$

while $M > N_e/2$. Here, we stress that these conditions are disjoint and cover all $M \leq N/2$, i.e., this class of limit cycles is uniquely determined by the number of particles if the initial state is general.

Fig. 9 shows an example of the flow of each part versus the number of particles. One observes that points are completely fitted with the theoretically obtained curves.

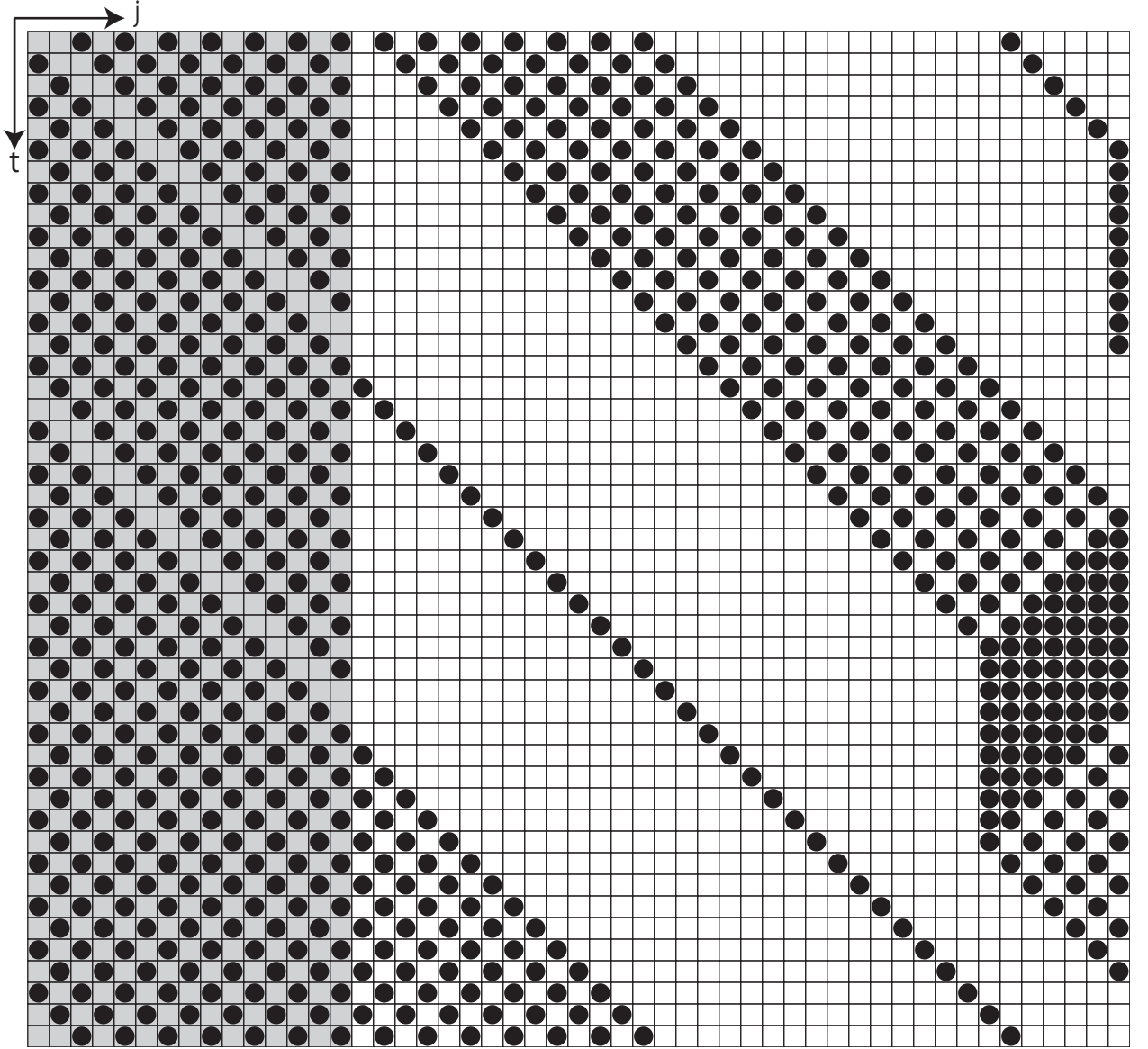


FIG. 8: An orbit of limit cycle class 1-2 for 1 period. The parameters are $N_e = 15$, $N_i = 36$, $M = 15$.

G. Special limit cycles

We next discuss the limit cycles arisen by special initial states. If the number of particles is sufficiently small, one can construct the system which provides such cycles. For example, we consider the system where parameters N_e and N_i are satisfying $N_i = 2N_e - 2$ and the initial value is $u_1^0 = u_{N_e}^0 = 1$ and $u_j^0 = 0$ for other sites. The particle at $j = N_e$ cannot jump to $j = 1$ but moves to $j = N_e + 1$ because the other occupies there. Then, the particle at $j = N_e + 1$ moves along to the i -part and the other goes around the e -part several times. When the particle in the i -part arrives the end of the i -part ($j = N$), the other is at $j = N_e - 1$ and two particles are at $j = 1$ and $j = N_e$ in the next time, respectively, which is the same as the initial state. Therefore, in this case, not all particles are trapped in the e -part for this limit cycle even if M is less than $N_e/2$ against the discussion before. Such special parameters and initial values can be constructed by setting, for example, $N_i = qN_e - q + 1$ for $q = 1, 2, \dots$, $u_1^0 = u_{N_e}^0 = u_{N - N_e + 1}^0 = 1$ and $u_j^0 = 0$ for other sites. By numerical experiments, we observe that such special initial states are rare. However, they arise relatively often around $M = N_e/2$, which can be an indirect evidence of the phase shift by this point. Generally, it is very difficult problem to determine from initial states or construct such limit cycles for larger M because these special limit cycles are induced by very complicated and balanced interactions and so unstable that they are attracted by the stable limit cycles with a little perturbation such as shifting one of particles to the neighbor empty site. We

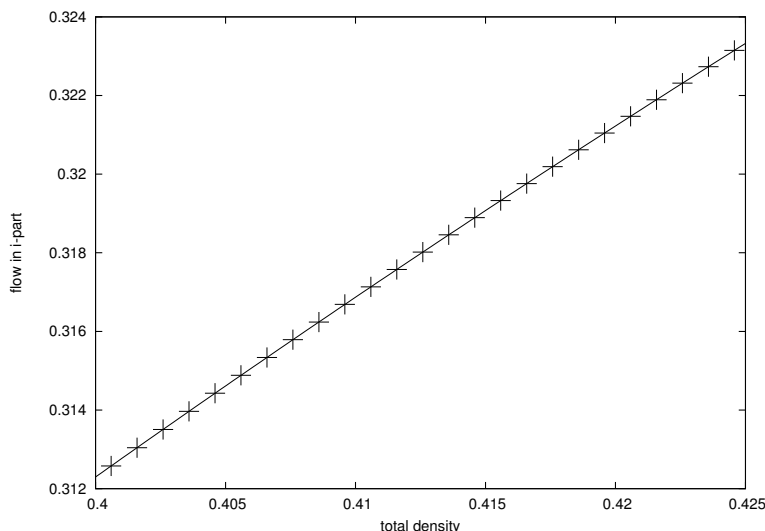


FIG. 9: A part of the graph of the flow in the i -part for $N_e = 301$, $N_i = 700$. Solid curves are drawn by the formulae and points are numerically calculated by definition (14).

finally note that such special initial states are not depend on the parity of N_e , differently from general limit cycles.

Fig. 10 is an example of such limit cycles. By observing these cycles, there are several notches appear in the e -part for each time. Due to the discussion above, the number of notches should be odd for the odd-length e -part and even for the even-length. It is very difficult to obtain the properties of these special limit cycles generally by the method to discuss general one like above because we have to prepare concrete limit cycles in this method. However, by numerical observation, one may classify these special limit cycles like general ones and the condition to attract is independent on that of general ones (See Fig. 6).

IV. CONCLUDING REMARKS

In this paper, we considered the dynamics for the simplified version of the path-preference model. We discuss the property for a class of limit cycles, which are raised by general initial states, for example, the conditions of system parameters where the orbit is attracted by one of them and the exact value of the flow in each part which depend only on the length of each part and the number of particles.

If one wants to apply to that for more complex lives, for example, humans, one should consider the original path-preference model which contains stochastic factors and several e - and i -parts. One can expect that the dynamics of the simplified model with the stochastic factor is the intermediate between the simplified model and the ECA rule 184 which correspond $p = 1$ and $p = 0$, respectively and the numerical results follows this expectation. One can expect that the number of the particles is also dominant for the dynamics of K e - and i -parts model with no stochastic factors and traffic jams occur at the end of i -parts as it increases. This may also be true by numerical experiments.

We should also remember that we employ the jump priority rule just because of the mathematical symmetry. If experiments follow the jump priority rule, our model is not applicable to this field. However, we can apply similar discussion with a little modification to predict the behavior of the system. For example, the aspect of the dynamics is the same as that in the jump priority rule when the number of particles is sufficiently small but it behaves similar to the ECA rule 184 when the number of particles is sufficiently large.

In the case including other factors, for example, regulation by the epigenetic modification or non-genetic bases to the path-preference model, the global behavior of the dynamics still keeps if these factors effect the dynamics locally.

We do not understand properties about special limit cycles, for example, the classification of such cycles or what initial states are attracted by the special limit cycles and their properties like general ones. These topics are only mathematically interesting but not real dynamics.

Recent measurements reveal that there are not few genes which contain only one exon. Among 24,875 human genes in RefGene (hg19) database, the number of genes which consist only one exon is estimated to be 985 and the lengths of these genes are ranging from 27 to 91672bp. This result indicates that the range of our result is broad in spite of its simplicity. If one can observe the transcription dynamics for one gene, we expect that our result helps to analyze it. It is also expected that the analysis in the paper can apply to other topics, for example, traffic flow models with

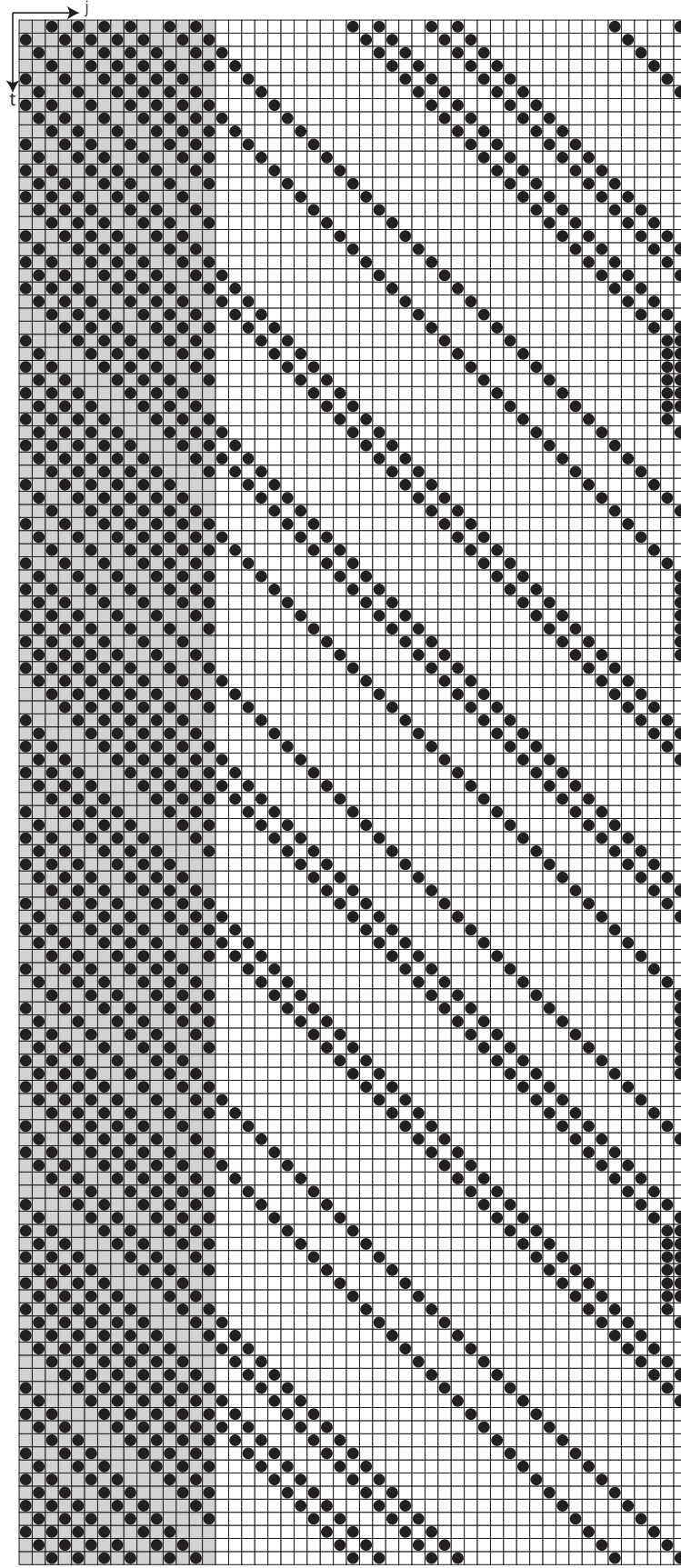


FIG. 10: An example of special limit cycles. The parameters are $N_e = 15$, $N_i = 36$, $M = 12$. The period of this limit cycle is 117 though that of general limit cycles should be 40. We also note that the general limit cycles also exist for this parameter.

several preferential paths.

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Appendix A: Mathematical implementation of simplest version

The number of moving particles is the same as (1) and that of jumping particles is expressed as

$$f(N_e, 1) = \min(u_{N_e}^t, 1 - u_1^t). \quad (\text{A1})$$

If $f(N_e, 1) = 1$, the number of transitional particle between two parts is

$$f(N_e, N_e + 1) = 0 \quad (\text{A2})$$

$$f(N, 1) = 0. \quad (\text{A3})$$

If $f(N_e, 1) = 0$, one has

$$f(N_e, N_e + 1) = \min(u_{N_e}^t, 1 - u_{N_e+1}^t) \quad (\text{A4})$$

$$f(N, 1) = \min(u_N^t, 1 - u_1^t). \quad (\text{A5})$$

Then, the time evolution rule is written in

$$u_1^{t+1} = u_1^t + f(N_e, 1) + f(N, 1) - f(1, 2) \quad (\text{A6})$$

$$u_{N_e}^{t+1} = u_{N_e}^t + f(N_e - 1, N_e) - f(N_e, 1) - f(N_e, N_e + 1) \quad (\text{A7})$$

and

$$u_j^{t+1} = u_j^t + f(j - 1, j) - f(j, j + 1) \quad (\text{A8})$$

for other sites.

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